UNIVERSITY OF SASKATCHEWAN
ELECTRICAL ENGINEERING 455.3

10

Assignment Quiz 7
November 21, 2001

Instructor: B.L. Daku
Time: 15 minutes
Aids: None

Name:
Student Number:

1. When the input to an LTI system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + 2^n u[-n-1],$$

the corresponding output is

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n].$$

(a) Find the system function $H(z)$ of the system. Plot the pole(s) and zero(s) of $H(z)$ and indicate the region of convergence.

(b) Find the impulse response $h[n]$ of the system.

(c) Write a difference equation that is satisfied by the given input and output.

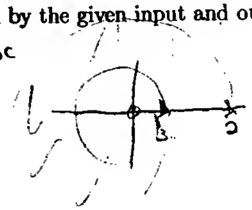
(d) Is the system stable? Is it causal?

ROC

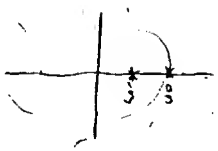
a) $X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{-1}{1-2z^{-1}}$

$= \frac{\cancel{1}-2z^{-1} \cancel{-1} + \frac{1}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$

$= \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$



$y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} + \frac{-5}{1-\frac{2}{3}z^{-1}}$



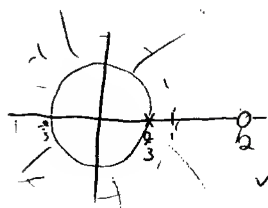
$= \frac{\cancel{5} - \frac{10}{3}z^{-1} \cancel{-5} + \frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}$

$= \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}$

$H(z) = \frac{\cancel{-\frac{5}{3}z^{-1}}}{\cancel{(1-\frac{1}{3}z^{-1})}(1-\frac{2}{3}z^{-1})} \cdot \frac{\cancel{(1-\frac{1}{3}z^{-1})}(1-2z^{-1})}{\cancel{-\frac{5}{3}z^{-1}}}$

$\boxed{H(z) = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}}}$

ROC $\boxed{|z| > \frac{2}{3}}$



b) $H(z) = \frac{1}{\ldots} - \frac{2z^{-1}}{\ldots}$

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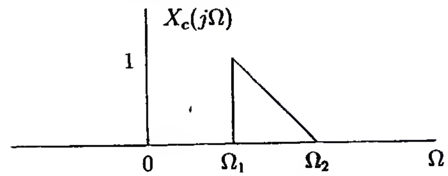
1. A complex-valued continuous-time signal, $x_c(t)$, has the Fourier transform shown in the following figure. The signal is sampled to produce the sequence $x[n] = x_c(nT)$.

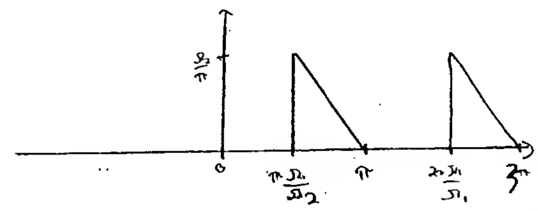


(a) Sketch the Fourier transform, $X(e^{j\omega})$, of the sequence $x[n]$ for $T = \pi/\Omega_2$.

(b) What is the lowest sampling frequency that can be used without incurring any aliasing distortion, i.e., so that $x_c(t)$ can be recovered from $x[n]$. Show your work. Sketch $X(e^{j\omega})$ using this sampling frequency.

(c) Draw the block diagram of a system that can be used to recover $x_c(t)$ from $x[n]$ if the sampling rate is greater than or equal to the rate determined in part b). Assume that (complex) ideal filters are available.

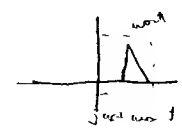
a) $T = \frac{\pi}{\Omega_2}$

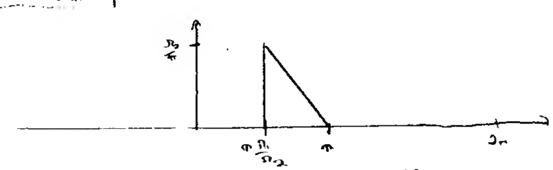


b) $\frac{1}{T} \geq \frac{\Omega_2}{2\pi}$ $\quad \Omega_s \geq \frac{2\pi}{T}$

$\boxed{f_s \geq \frac{\Omega_2}{2\pi}}$

$\Omega_s \geq \Omega_2 - \Omega_1$





c)